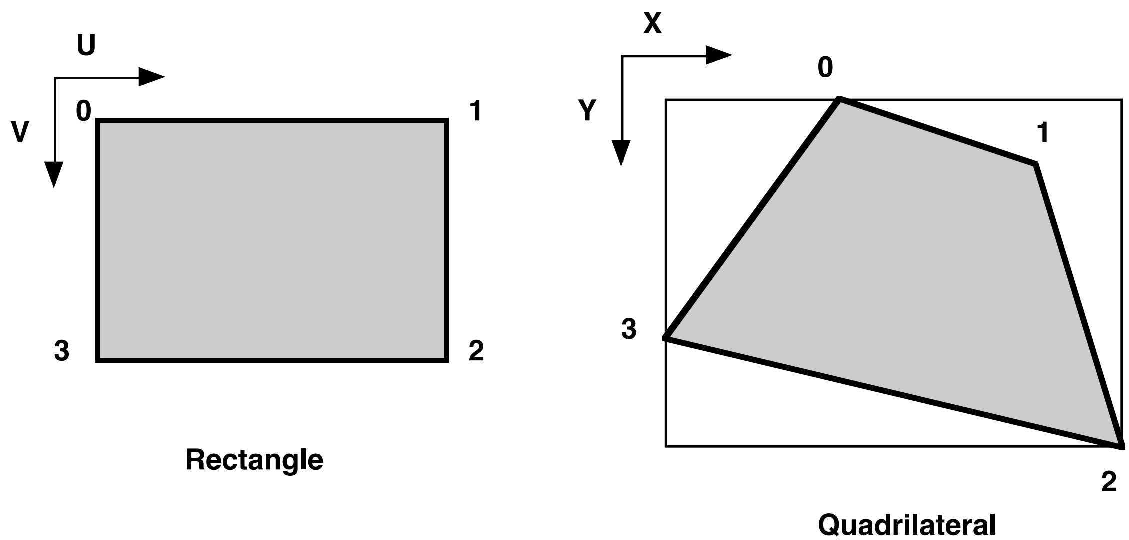
# Four Corner Image Warping

Consider the following diagram:




We desire to warp the rectangle into the quadrilateral.

There are three well-know ways to do this.

1) 1.5 Order Polynomial Transformation
2) Bilinear Transformation
3) Perspective Transformation

Each on permits a set of transformation equations involving 8 coefficients to be solved from four pairs of conjugate coordinates, namely the (U,V) corners of the rectangle and the (X,Y) corresponding vertices of the quadrilateral.

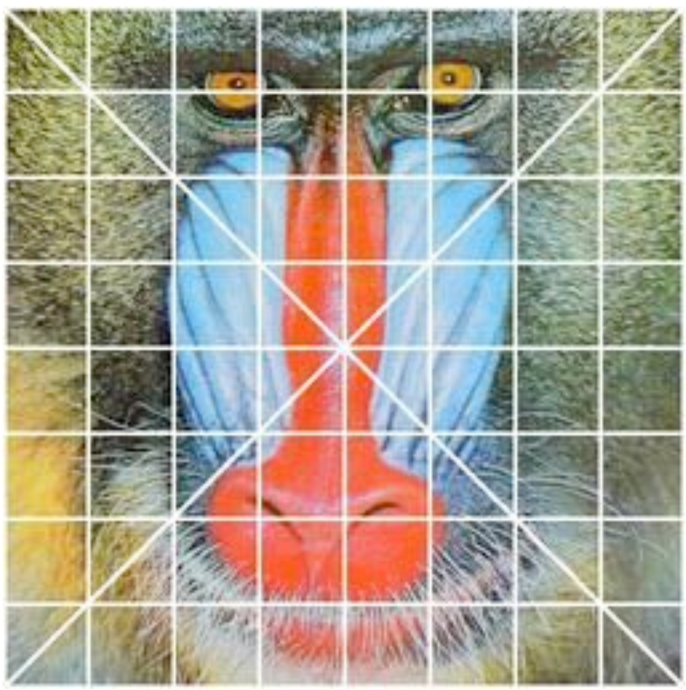
In general image warping is done as a reverse transformation. Thus we sequence through every (X,Y) pixel in the quadrilateral and find its corresponding (U,V) coordinate in the rectangle. Then interpolate the neighboring values and put the resultant value at the specified (X,Y) location in the quadrilateral.

This is expressed in general by the following equation:

1a) $U = \mathrm{F}(X,Y)$

1b) $V = \mathrm{G}(X,Y)$

Consider a rectangle image of size 256 x 256, which is the mandril image with a grid superimposed.



For the quadrilateral, lets use the following corresponding vertices as defined by the diagram at the top.

Pt0=(52,0)
Pt1=(228,46)
Pt2=(255,229)
Pt3=(0,246)

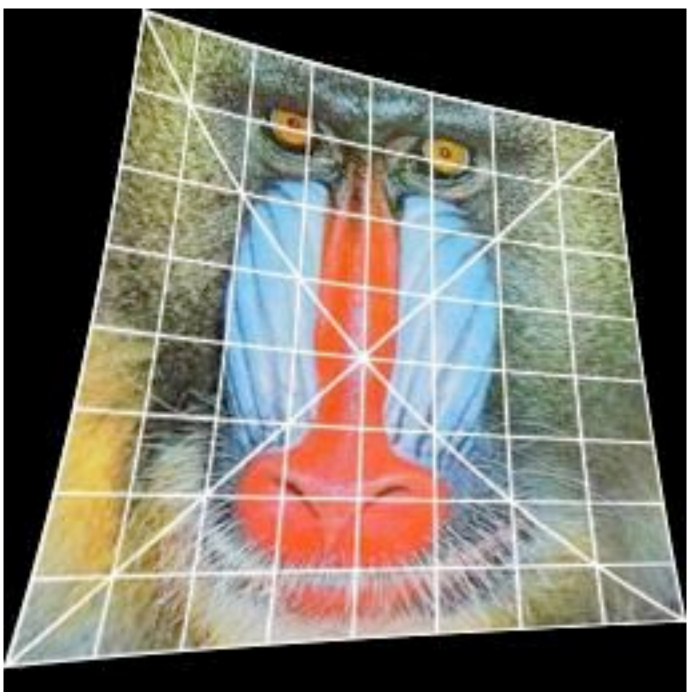
Method 1: 1.5 Order Polynomial

Equation 1) is expressed in the following polynomial form which is linear, but includes the cross term XY.

2a) $U = a_0 + a_1 X + a_2 Y + a_3 XY$

2b) $V = b_0 + b_1 X + b_2 Y + b_3 XY$

We can solve for the (a,b) coefficients by substituting the four sets of conjugate control points into this equation. Once they are obtained, we can apply the 1.5 order polynomial warp of the rectangle to the quadrilateral. This produces the following image:



We observe that none of the straight lines are preserved from the rectangle. That is in the quadrilateral they all curve including the boundary edges.

Method 2: Bilinear Transformation

What we want here is actually the inverse equation the following:

3a) $X = a_0 + a_1U + a_2V + a_3UV$

3b) $Y = b_0 + b_1U + b_2V + b_3UV$

To do that, we solve for U from equation 3a), namely,

4) $U = (X - a_0 - a_2V)/(a_1 + a_3V)$

Substituting this into equation 1b), rationalizing the denominators and combining like powers of V, we find the following quadratic equation that must be solved to get V

5) $AV^2 + BV + C = 0$

where

$A = (b_2a_3 - b_3a_2)$

$C = C_1 + (b_1X - a_1Y)$

where

$C_1 = (b_0a_1 - b_1a_0)$ is a constant independent of X or Y

$B = B1 + (b_3X - a_3Y)$

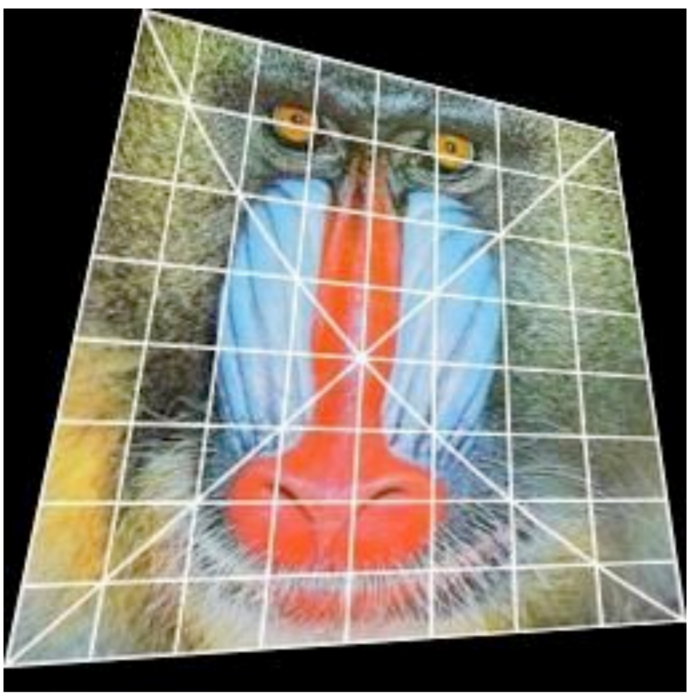
where

$B1 = (b_0 a_3 - b_3 a_0) + (b_2 a_1 - b_1 a_2)$ is a constant independent of X or Y

The solution to 5) is of course,

6) $$V = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Applying the bilinear transformations from equations 4) and 6) to the mandril image, results in:



We observe here that lines parallel to the coordinate axes of the input rectangle image remain straight lines in the output quadrilateral image. Thus the quadrilateral boundary edges are straight and the horizontal and vertical grid lines from the rectangle remain straight lines in the quadrilateral. However, one or both diagonals may end up curved depending upon the transformation. They are not necessarily preserved as straight line.

## Method 3: Perspective Transformation

The equations describing the perspective transformation are a ratio of linear polynomials. They may be expressed as:

7a) $$U = \frac{a_0 + a_1 X + a_2 Y}{1 + c_1 X + c_2 Y}$$

7b) $$V = \frac{b_0 + b_1 X + b_2 Y}{1 + c_1 X + c_2 Y}$$

Applying the perspective transformations from equations 7) to the mandril image, results in:

We note here that all straight lines in the rectangle are preserved in the quadrilateral. This includes the diagonals.