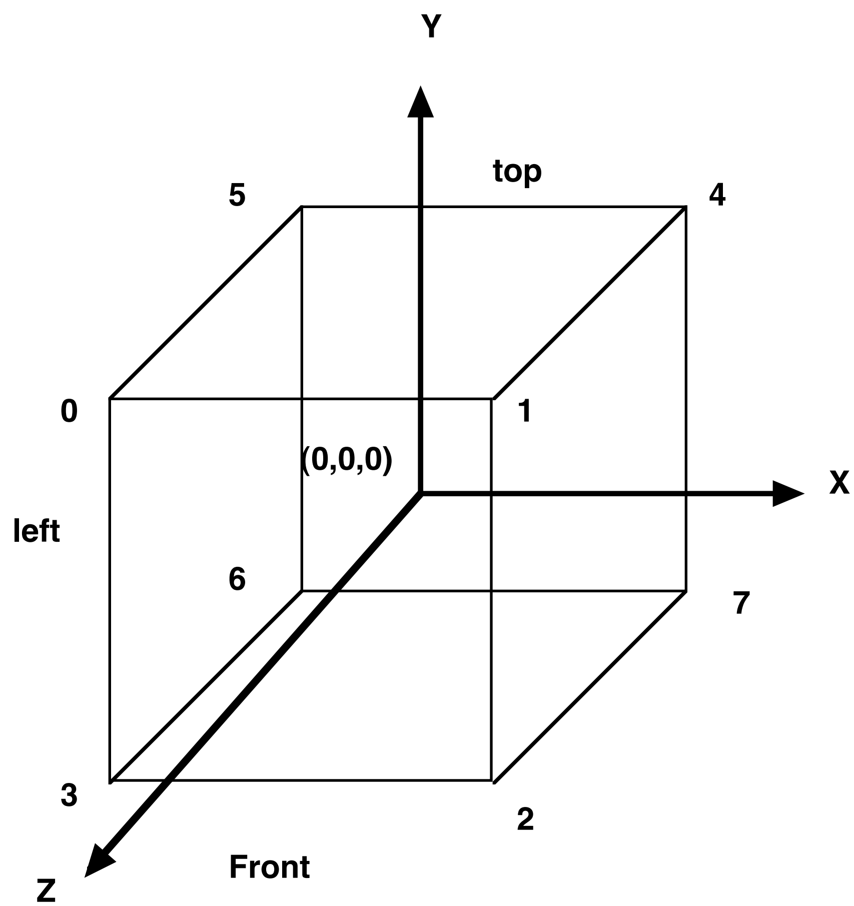
3DBOX

Consider a box located with centroid at the world coordinate origin. Let the coordinate axes be defined so that Z points towards the viewer, X and Y point to the right and up, respectively, as seen by the viewer. Define vertices and surfaces so that the vertices for each surface are ordered clockwise as seen from the outside (for later back facing test).

S1: 0,1,2,3 (Z positive; front)
S2: 5,0,3,6 (X negative; left)
S3: 5,4,1,0 (Y positive; right)
S4: 4,5,6,7 (Z negative; back)
S5: 1,4,7,2 (X positive; right)
S6: 7,6,3,2 (Y negative; bottom)



Let the camera be placed at Zc=f from the box centroid (0,0,0) in world coorndinates, i.e. (0,0,Zc) = (0,0,f), where f = largest box half diagonal (i.e. max of all box vertices from its centroid) = maxdist. Thus, including a perspective exaggeration factor, pef, we have

f = maxdist/(2*tan(pef*fov/2), where fov=53 deg

Now the perspective equations (in 3-D) are defined as (x,y,f) = M (X',Y',Z'), where the camera orientation matrix M is the identity matrix but with M22=-1, because the camera is looking straight down along -Z. Thus, we have a reflection transformation relative to the ground plane coordinates.

M0 = (1,0,0)
M1 = (0,1,0)
M2 = (0,0,-1)

Now we want to rotate the ground points corresponding to the picture corners. Thus the basic rotation is (X',Y',Z') = R (X,Y,Z), where R is the rotation matrix involving pan, tilt and roll. Thus

(x,y,f) = M (X',Y',Z'-Zc) = M (X',Y',Z') - M (0,0,f)

or

(x,y,f) = MR (X,Y,Z) - M (0,0,f)

But we need to allow for offset of the output coordinates and conversion from (x,y,f) to (u,v,1), where v increases downward. Thus (x,y,f)=A(u,v,1) where (x=u,y=-v). Thus

A0 = (1,0,0)
A1 = (0,-1,0)
A2 = (0,0,f)

Thus the forward transformation from world coordinates to output picture coordinates becomes,

$A\ (u,v,1) = MR\ (X,Y,Z) - M\ (0,0,f)$

Or inverting A as A^{-1}

$(u,v,1) = A^{-1}MR\ (X,Y,Z) - A^{-1}M\ (0,0,f)$

But we will merge $A^{-1}M$ into Aim. As $AA^{-1}=I$ identity matrix, it is easy to see that

$A^{-1}0 = (1,0,0)$
$A^{-1}1 = (0,-1,0)$
$A^{-1}2 = (0,0,1/f)$

So that Aim becomes,

Aim0 = (1,0,0)
Aim1 = (0,-1,0)
Aim2 = (0,0,-1/f)

Thus,

(u,v,1) = AimR (X,Y,Z) + (0,0,1)

O = PW + (0,0,1)

where

P = AimR
O = (u,v,1)
W = (X,Y,Z)