ROTATE3D

The four picture corners are:

UL = (0,0)
UR = (NS, 0)
BR = (NS, NL)
BL = (0, NL)

where

NS = width
NL = height

Now, treat the picture as if it was on a vertical rectangle centered at and oriented parallel to the Y=0 plane.

Let the coordinate units be pixels, so that the four corner points become:

UL = (X, Y, Z) = (-NS2, 0 , NL2)
UR = (X, Y, Z) = (NS2, 0, NL2)
BR = (X, Y, Z) = (NS2, 0, -NL2)
BL = (X, Y, Z) = (-NS2, 0, -NL2)

where

NS2 = (NS – 1)/2 defines the center horizontal coordinate
NL2 = (NL – 1)/2 defines the center vertical coordinate

Treat the perspective camera as located a distance f = focal length (in equivalent pixel units) behind the picture looking straight at the picture center. Thus Yc = -f, where F is determined by the fov (field of view) defined by the diagonal dimension of the image.

tan(fov/2) = sqrt (NS^2 + NL^2) / (2 * f)

or

$f = NS / (2 * \tan (FOV/2))$

where fov = the equivalent fov for 35mm picture frame whose dimensions are 36mm x 24mm. Thus

$fov = 180 * atan(36/24) / pi$ (which is approx. 56 degrees)

Then rotate the picture rectangle corner points by the pan, tilt and roll combined rotation matrix, R.

Let the 3 rotation angles be defined in the following order as:

Pan = right hand positive rotation of points about Z axis
Tilt = right hand negative rotation of points about X axis
Roll = right hand positive rotation of points about Y axis

Then the combined rotation matrix becomes:

$R00 = (croll * cpan) + (sroll * stilt * span)$
$R01 = (croll * span) - (sroll * stilt * cpan)$
$R02 = (sroll * ctilt)$
$R10 = - (ctilt * span)$
$R11 = (ctilt * cpan)$
$R12 = (stilt)$
$R20 = - (sroll * cpan) + (croll * stilt * span)$
$R21 = - (sroll * span) - (croll * stilt * cpan)$
$R22 = (croll * ctilt)$

where the leading s or c means sin or cos.

Then perspectively project the rotated points to the camera.

Because $Xc = Zc = 0$, the perspective equations become:

$Xp/F = (X - Xc) / (Y - Yc) \Rightarrow Xp = f * X / (Y + f)$
$Zp/F = (Z - Zc) / (Y - Yc) \Rightarrow Zp = f * Z / (Y + f)$

But we need to convert from Xp to S (sample) and Zp to L (line) such that S,L are at the top left rather than the center and L increases downward.

Xp = -NS/2 + S   where NS = number of samples (width)
Zp = NL/2 – L   where NL = number of lines (height)

Thus the perspective equations become

S = ((f * X) / (Y + f)) + NS2
L = NL2 - ((f * Z) / (Y + f))

Now project the four rotated corner points using these equations.

Finally use the four projected points with the four original picture points to feed to the IM function -distort perspective.