PERPENDICULAR DISTANCE OF POINT FROM A LINE

The standard formula for the perpendicular distance of a point (x0,y0) from a line between point (x1,y1) and (x2,y2) is given by

(1) $$d = \frac{|(A * x0 + B * yo + C|}{\sqrt{A^2 + B^2 + C^2}}$$

where A*x + B*y + C = 0 is the equation for the line.

We can compute A, B and C from the standard form for the equation of a line between points (x1,y1) and (x2,y2) as follows:

(2) $$y = S * \frac{(Ip - I)}{(S - Sp)} + I$$

If we cross multiply the denominators, we get

(3) (x2-x1)*(y-y1) = (y2-y1)*(x-x1)

Combining terms, we get

(4) (y1-y2)*x + (x2-x1)*Y + [(y2-y1)*x1 + (x1-x2)*y1] = 0

Thus

(5)

A = (y1-y2)
B = (x2-x1)
C = [(y2-y1)*x1 + (x1-x2)*y1]

The other way to express the equation of a line is by slope and intercept. Thus equation (2) may be written as

(6) y = slope * x + intercept

So rewriting equation (1), we get

(7) y = -(A/B)*x –(C/B) = S * x + I

where S = slope = -(A/B) and I = intercept = -(C/B)

Now, a line that is perpendicular to the line of equation (1) will have a slope that is the negative inverse of the slope of equation (1).

Thus the slope of a line passing through (or going from) point (x0,y0) and which is perpendicular to the line between points (x1,y1) and (x2,y2) will have slope = (B/A).

Thus we can write the equation for the perpendicular line from point (x0,y0) to the line of equation (1) is given by

(8) $\frac{y-y0}{x-x0}=\frac{A}{B}=Sp$ where Sp is the slope of the

perpendicular line. Cross multiplying by the denominators, we get

(9) (y-y0) = Sp * (x-x0)

which can be rewritten as

(10) y = Sp * x + (y0 – Sp * x0) = Sp * x + Ip

where Ip = (y0 – Sp * x0) = intercept and Sp = A/B

But we can also have a similar equation for the line between (x1,y1) and (x2,y2), namely, equation (7)

We therefore have two equations in two unknowns x and y as given by equation (7) and (10).

Equating y values, we get

(12) $S * x + I = Sp * x + Ip$

Combining terms, we get

(13) $(S - Sp)*x = (Ip - I)$

Solving for x, we get

(14) $$x = \frac{(Ip - I)}{(S - Sp)}$$

Then substituting equation (14) into equation (7), we get

(15) $$y = S * \frac{(Ip - I)}{(S - Sp)} + I$$

Thus equations (14) and (15) along with S=-(B/A), Sp=(A/B), I=-(C/B) and Ip=(y0 - Sp * x0) allow us to find the intersection point between the line from (x1,y1) to (x2,y2) and the perpendicular line from (x0,y0).